Gravitation conservation of energy and superluminal signals

Abstract

Gravitation is shown not to satisfy conservation of energy and having no superluminal signals.

We restrict to theories of gravitation where the only constants with dimension are $c$ and $G$. Units are chosen so that $c = G = 1$. Let $x, y, z$ be coordinates of space and consider a particle $A$ at the origin of total energy $M$. Let $E_R$ be the energy in the gravitational field of $A$ from the set of points with $r > R$ where $x^2 + y^2 + z^2 = r^2$. Since $E_R = 0$ when $M = 0$ and $E_R$ has dimensions of energy we have for small $M/R$

$$E_R = M \left[a_0 + a_1 \frac{M}{R} + a_2 \frac{M^2}{R^2} + a_3 \frac{M^3}{R^3} + \cdots\right]$$

(1)

where the $a_k$ are dimensionless constants. Now $E_R$ approaches zero as $1/R$ approaches zero hence $a_0 = 0$.

Let $\gamma$ be a photon emitted at infinity moving along the $x$ axis towards the origin. Let $E$ be the energy of $\gamma$ at infinity. As $\gamma$ moves towards the origin it gains energy. The energy that $\gamma$ gains on reaching $r = R$ is zero when $ME$ is zero and approaches zero as $1/R$ approaches zero so for small $M/R$ and $E/R$ this gain is

$$\frac{ME}{R} \left[b_0 + b_1 \frac{M}{R} + b_2 \frac{E}{R} + b_3 \frac{M^2}{R^2} + b_4 \frac{ME}{R^2} + \cdots\right]$$

(2)

where the $b_k$ are dimensionless constants. By Einstein\(^1\) we have $b_0 = 1$. Signals are assumed not to travel faster than light so the energy gained by $\gamma$ on reaching $r = R$ must come from $E_R$. Energy can not come from $A$ at the origin since there is no signal in advance of $\gamma$. By Eqs. (1), (2), and conservation of energy

$$\frac{ME}{R} \left[1 + b_1 \frac{M}{R} + b_2 \frac{E}{R} + b_3 \frac{M^2}{R^2} + \cdots\right] < \frac{M^2}{R} \left[a_1 + a_2 \frac{M}{R} + a_3 \frac{M^2}{R^2} + \cdots\right].$$

(3)

This does not hold for small enough $M$ and $E$.

References