A paradox for an accelerating frame of reference

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Abstract: Do clocks equidistant from the axis of rotation of a constant angular accelerating frame of reference go at the same rate?

Résumé: Plusières horloges placées à distances égales de l’axe de rotation d’un référentiel en accélération angulaire constante ont-elles la même cadence?

Key words: Rotating; Acceleration; Potential
Let $\mathcal{F}$ be an inertial frame of reference with coordinates $x, y, z, t$ and $\mathcal{F}'$ a frame of reference with coordinates $x', y', z', t'$ rotating about the $z$ axis. $\mathcal{F}$ and $\mathcal{F}'$ having common origins. Let $\theta$ be the angle between the $x$ and $x'$ axis. On the $x'y'$ plane of $\mathcal{F}'$ let there be a regular polygon centred at the origin with side $s$ and radius $R$ of circumscribed circle. Let $V'_1$ be a vertex of the polygon and let $V'_2$ be the first vertex counterclockwise from $V'_1$. Let $\ddot{\theta} > 0$ be constant and initially $\dot{\theta} = 0$. For a small interval of time and $R \gg s$ with respect to $\mathcal{F}$ the vertices $V'_1$ and $V'_2$ will initially be at rest and will be approximately moving along a straight line with constant acceleration $\ddot{\theta}R$. Let $\Delta t$ be a small interval of time measured by a clock at rest with respect to $\mathcal{F}$. Let $\Delta t'_2$ be this time interval measured with respect to $\mathcal{F}'$ by a clock at rest at $V'_2$ and define similarly $\Delta t'_1$. Using the equivalence principle the ratio of these intervals is approximately [1], p.80,

$$\frac{\Delta t'_2}{\Delta t'_1} = 1 - \frac{\Delta \phi}{c^2} = 1 - \frac{\ddot{\theta}Rs}{c^2} \quad (1)$$

where $\Delta \phi$ is the difference in potential between $V'_2$ and $V'_1$ so $\Delta t'_2 < \Delta t'_1$ and at the $n$th vertex $\Delta t'_{n+1} < \Delta t'_n$. If there are $N$ vertices then at the $N$th vertex $\Delta t'_1 < \Delta t'_N$. Consequently

$$\Delta t'_1 < \Delta t'_N < \Delta t'_{N-1} < \cdots < \Delta t'_2 < \Delta t'_1 \quad (2)$$

which does not hold.

References