Energy of a system of particle and photon is not conserved

Karl De Paepe∗

Abstract
We consider a photon moving from infinity towards a point mass along a fixed line containing the mass. We show the system does not satisfy conservation of energy.

1 Introduction
We restrict to a gravitation that has only constants $c$ and $G$ with dimension. Units are chosen so that $c = G = 1$. Let $x, y, z$ be coordinates of space and consider a particle $A$ on the $x$ axis. Let $\gamma$ be a photon moving along the $x$ axis from infinity towards $A$. The photon being considered as a classical particle with zero rest mass. When $\gamma$ is at infinity let $A$ be at rest at the origin and have total energy $M$ and $\gamma$ have energy $E$.

2 Energy gain function
Let the function $W(M, E, R, h)$ represent the amount of energy $\gamma$ gains on moving from $x = R + h$ to $x = R$. Using dimensional analysis there is then a real valued function $F$ of the dimensionless variables $M/R, E/R,$ and $h/R$ such that

$$W(M, E, R, h) = \frac{MEh}{R^2}F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right)$$  \hspace{1cm} (1)

∗k.depaep@utoronto.ca
For small $M/R$, $E/M$, and $h/R$ the function $F(M/R, E/R, h/R)$ is approximately one hence $F(0, 0, 0) > 0$.

### 3 Bound on energy gain

Since $A$ has only an amount $M$ of energy by conservation of energy $\gamma$ cannot gain more than $M$ amount of energy from $A$ hence

$$W(M, E, R, h) = \frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) < M$$

so

$$\frac{Eh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) < 1$$

Consider the subset of real numbers

$$\left\{ \frac{Eh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) : E > 0 \right\} \subset \mathbb{R}$$

By (3) this set is bounded above by one. There is then by dimensional analysis a real valued function $B(M/R, h/R)$ such that

$$\sup_{E} \left\{ \frac{Eh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) : E > 0 \right\} = \frac{h}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) \leq 1$$

Consider the subset of real numbers

$$\left\{ B\left(\frac{M}{R}, \frac{h}{R}\right) : R > NM \right\} \subset \mathbb{R}$$

where $N$ is a large natural number. For small nonzero $M/R$ and $E/M$ since $W(M, E, R, h) > 0$ we have $B(M/R, h/R) > 0$. As a result the set (6) is bounded below by zero. For $h \neq 0$ we have by dimensional analysis there is then a real valued function $f$ such that

$$\inf_{R} \left\{ B\left(\frac{M}{R}, \frac{h}{R}\right) : R > NM \right\} = f\left(\frac{M}{h}\right)$$

By (5)

$$\frac{h}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) \leq 1$$
hence $B(M/R, h/R)$ exists at $M = 0$ so $f(M/h)$ must exist at $M = 0$. For $M \neq 0$ we have by dimensional analysis there is a real valued function $g$ such that

$$\inf_{R} \left\{ B\left(\frac{M}{R}, \frac{h}{R}\right) : R > NM \right\} = g\left(\frac{h}{M}\right) \tag{9}$$

The set of real numbers

$$\left\{ \frac{\partial W}{\partial h}(M, E, R, 0) = \frac{ME}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, 0\right) : E > 0 \right\} \tag{10}$$

will be bounded above otherwise (5) would not hold. Consequently

$$B\left(\frac{M}{R}, 0\right) = \sup_{E} \left\{ \frac{E}{R} F\left(\frac{M}{R}, \frac{E}{R}, 0\right) \right\} \tag{11}$$

exists hence $g(h/M)$ must exist at $h = 0$. For $Mh \neq 0$ we have $f(M/h) = g(h/M)$ so by a power series of $f$ and $g$ we must have $f$ and $g$ are constant functions. By (8) $B(M/R, h/R) \to 0$ as $h \to \infty$ so the constant value must be zero. Now $B(M/R, h/R) > 0$ so we can conclude $B(M/R, h/R) \to 0$ as $R \to \infty$.

\section*{4 Contradiction}

Define the function

$$C(M, h, R) = RB\left(\frac{M}{R}, \frac{h}{R}\right) \tag{12}$$

We have since $B(M/R, h/R) \to 0$ as $R \to \infty$ that $C(MR/r, hR/r, R) \to 0$ as $r \to \infty$. By (5) and (12)

$$\frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq \frac{Mh}{R^2} B\left(\frac{M}{R}, \frac{h}{R}\right) = \frac{Mh}{R^2} C(M, h, R) \tag{13}$$

so

$$EF\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq C(M, h, R) \tag{14}$$

Substitute $MR/r$ for $M$ and $hR/r$ for $h$ in this inequality and let $r \to \infty$ gives since $C(MR/r, hR/r, R)$ goes to zero and $E > 0$ that

$$F\left(0, \frac{E}{R}, 0\right) \leq 0 \tag{15}$$

We stated before that $F(0, 0, 0) > 0$. This contradicts (15).
5 Alternative argument

This section is incomplete. Let $h = E^2/R$ and let $R \to 0$ keeping $M/R$ fixed. Consequently $M \to 0$. We expect as the mass of $A$ goes to zero the amount of energy $\gamma$ can gain goes to zero. In the limit this would correspond to $\gamma$ moving from infinity to the origin with $A$ having no mass hence

$$
\lim_{R \to 0} W \left( M, E, R, \frac{E^2}{R^2} \right) = \lim_{R \to 0} \frac{M}{R} E \frac{E^2}{R^2} F \left( \frac{M}{R}, \frac{E}{R}, \frac{E^2}{R^2} \right) = 0 \quad (16)
$$

keeping $M/R$ fixed. This implies

$$
\lim_{E \to \infty} E^2 F \left( \frac{M}{R}, \frac{E}{R}, \frac{E^2}{R^2} \right) = 0 \quad (17)
$$

Consequently the two functions of $E$

$$
E^2 F \left( \frac{M}{R}, \frac{E}{R}, \frac{E^2}{R^2} \right) \quad EF \left( \frac{M}{R}, \frac{E}{R}, \frac{E^2}{R^2} \right) \quad (18)
$$

have maxima. In the following we show that (16) does not hold. This will imply that $\gamma$ can gain more energy than the mass of $A$ for small enough mass of $A$.

6 Conclusion

On considering a system of a photon moving from infinity towards a point mass along a fixed line containing the mass we showed energy is not conserved. The energy of the system when the photon is at infinity is $M + E$. The energy gain of the photon can become larger than $M$. The total energy of the system would then be larger than $M + E$. This suggests for systems with forces acting on moving particles that the total energy of the system increases.