Conservation of energy and gravitation for a photon moving towards a point mass

Karl De Paepe

Abstract
We consider a photon moving from infinity towards a point mass along a fixed line containing the mass. We assume the energy the photon gains on moving between any two points increases with the energy the photon has at infinity and show a gravitation with only constants $c$ and $G$ with dimension does not satisfy conservation of energy.

1 Introduction

We restrict to a gravitation that has only constants $c$ and $G$ with dimension. Units are chosen so that $c = G = 1$. Let $x, y, z$ be coordinates of space and consider a particle $A$ on the $x$ axis. Let $\gamma$ be a photon moving along the $x$ axis from infinity towards $A$. The photon being considered as a classical particle with zero rest mass. When $\gamma$ is at infinity let $A$ be at rest at the origin and have total energy $M$. Let $E$ be the energy of $\gamma$ at infinity.

2 Energy gain function

As $\gamma$ moves towards $A$ it gains energy from $A$. Starting from positive $x$ infinity the amount of energy $\gamma$ gains on moving from an $x$ value of $R + h$ to an $x$ value of $R$ will be a function of $M, E, h,$ and $R$. Since $c$ and $G$ are the only constants with dimension there is then a dimensionless function $F$ of

* k.depaep@utoronto.ca
the dimensionless variables $\frac{M}{R}$, $\frac{E}{R}$, and $\frac{h}{R}$ such that we can write the energy gain as

$$\frac{M E h}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right)$$

(1)

For small $\frac{E}{M}$ and $\frac{M}{R}$ the amount of energy $\gamma$ gains on moving from infinity to $R$ is by [1] approximately $\frac{M E}{R}$. For small $\frac{E}{M}$, $\frac{M}{R}$, and $\frac{h}{R}$ we have (1) is approximately $\frac{M E h}{R^2}$.

3 Bound on energy gain

By conservation of energy

$$\frac{M E h}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq M$$

(2)

As a consequence of this bound there is then a dimensionless function $B\left(\frac{M}{R}, \frac{h}{R}\right)$ such that

$$\sup_E \left\{ \frac{M E h}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \right\} = \frac{M h}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) \leq M$$

(3)

For small $\frac{E}{M}$, $\frac{M}{R}$, and $\frac{h}{R}$ since (1) is approximately $\frac{M E h}{R^2} > 0$ we have $B\left(\frac{M}{R}, \frac{h}{R}\right) > 0$. Consequently we can define

$$b = \inf_{R > R_0} \left\{ B\left(\frac{M}{R}, \frac{h}{R}\right) \right\}$$

(4)

where $R_0$ is chosen so that $\frac{M}{R_0}$ and $\frac{h}{R_0}$ are small. We have $b \geq 0$.

4 $b = 0$

In this section we make an assumption and use conservation of energy to show $b = 0$. We now make the assumption that (1) is an increasing function of $E$. Consequently the more energy $\gamma$ has at infinity the more energy it gains on moving from $R + h$ to $R$.

The amount of energy $\gamma$ gains on moving from $R + (N + 1)h$ to $R$ is the amount of energy $\gamma$ gains on moving from $R + (N + 1)h$ to $R + Nh$ plus the
amount of energy $\gamma$ gains on moving from $R + Nh$ to $R + (N - 1)h$ and so on. For $\gamma$ having large energy at infinity this is approximately

$$\sum_{n=0}^{N} \frac{Mh}{R + (N - n)h} B\left(\frac{M}{R + (N - n)h}, \frac{h}{R + (N - n)h}\right) \geq \sum_{n=0}^{N} \frac{Mhb}{R + (N - n)h}$$

(5)

where $R > R_0$. It follows by (3) and the assumption that this approximation becomes better as the energy $\gamma$ has at infinity becomes larger. If $b > 0$ the sum becomes unbounded as $N \to \infty$ so for some $N$ the sum would become larger than $M$ hence the energy $\gamma$ gains would be larger than $M$ violating conservation of energy. We must have $b = 0$.

5 Contradiction

Since $B\left(\frac{M}{R}, \frac{h}{R}\right) > 0$ for $R > R_0$ and $b = 0$ it follows there must be a sequence $\{R_k\}$ where $R_k \to \infty$ as $k \to \infty$ such that $B\left(\frac{M}{R_k}, \frac{h}{R_k}\right) \to 0$ as $k \to \infty$. Define the function

$$C(M, h, R) = RB\left(\frac{M}{R}, \frac{h}{R}\right)$$

(6)

We have $C(M_k, h_k, R) \to 0$ as $k \to \infty$ where $M_k = \frac{MR_k}{R}$ and $h_k = \frac{hR_k}{R}$. By (3) and (6)

$$\frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq \frac{Mh}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) = \frac{Mh}{R^2} C(M, h, R)$$

(7)

or

$$E F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq C(M, h, R)$$

(8)

Substitute $M_k$ for $M$ and $h_k$ for $h$ in this inequality and let $k \to \infty$ gives since $M_k, h_k$, and $C(M_k, h_k, R)$ go to zero and $E > 0$ that

$$F\left(0, \frac{E}{R}, 0\right) \leq 0$$

(9)

As stated before for small $\frac{E}{R}, M, \frac{h}{R}$, and $\frac{h}{R}$ that (1) is approximately $\frac{MEh}{R^2}$. Comparing this with (1) we have $F(0, \frac{E}{R}, 0)$ for small $\frac{E}{R}$ is approximately one contradicting (9).
6 Conclusion

Assuming that the energy gain of a photon on moving from $R + h$ to $R$ increases as the energy the photon has at infinity increases it was shown that a gravitation with only constants $c$ and $G$ with dimension does not satisfy conservation of energy.

References